

## **APPENDIX B**

### **HAND CALCULATIONS OF UXO STAT [CALCULATOR]**

# GLOSSARY

Variable	Definition
$A$	area of total sector
$\rho_N$	density corresponding to specified UXO count in total sector
$k$	UXO count in subsector
$N$	specified UXO count in total sector
$p$	proportion of total sector investigated
$S$	area of subsector investigated
$\hat{U}$	expected UXO count in total sector
$\rho$	UXO density (per acre) in subsector
$\rho_s$	expected UXO density (per acre) in total sector

# Hand Calculations of UXO STAT

## Background

The U.S. Army Corps of Engineers, Engineering and Support Activity, Huntsville (HNC) has developed a computer based statistical methodology for determining the expected UXO density at ordnance sites. This program is called UXO Stat. The program contains a statistical methodology (based on the negative binomial distribution) that predicts the amount of OE at a site and the statistical confidence of the prediction. Another module in the tool allows the user to calculate the Minimum Discrimination Level based on the amount of land investigated and the size of the sector under investigation. The purpose of this effort is to evaluate the tool, ensure that the computer model predicts the appropriate amount of OE and provides the appropriate confidence level, and to evaluate the example generated by HNC to ensure its accuracy.

This report presents the results of UXO Stat hand calculations for two separate hypothetical examples and all modules of the tool. Except where noted, all hand calculations were verified using UXO Stat. The report also includes a review of an HNC provided example for accuracy and completeness.

## Report Organization

Appendix A provides the UXO Stat reference formulas used in the hand calculations. Appendix B provides the hand calculations for Case 1. Appendix C provides the hand calculations for Case 2. Each appendix is subdivided according to the modules of UXO Stat. Module 1 provides the inputs and initial "accounting" calculations. Module 2 calculates a probability based on total UXO count in the sector. Module 3 calculates the same probability, but based on the total UXO density in the sector. Module 4 calculates the total UXO count and density based on a specified probability. Module 5 calculates the minimum detection level for a sector when no UXO have been found. **Note: The author was unable to exercise Module 5 of the present version of UXO Stat. All Module 5 calculations presented in this report were performed by hand only.**

## **Comments on HNC Example:**

1. The example provided by HNC is numerically accurate.
2. The first sentence on the second page of the example states that the “results are self-explanatory.” It is suggested that an explanation be included for why UXO Stat projects a higher-than-uniform value for UXO count (and density) across the entire sector. It is the author’s understanding that this is due to the negative binomial/Bayesian use of prior information. A less statistical explanation might be useful. (The author provides an example in Appendix B where 0 UXO items are found in the subsector, but 9 UXO items are expected (projected) in the entire sector.)

## Appendix A: UXO Stat Reference Formulas

**Module 1:** The first four equations are “accounting” calculations for determining UXO counts and densities:

$$p = \frac{S}{A} \quad (1)$$

$$\rho = \frac{k}{S} \quad (2)$$

$$\hat{U} = \frac{k+1}{p} - 1 \quad (3)$$

$$\rho_s = \frac{\hat{U}}{A} \quad (4)$$

where,

A = area of total sector,

k = UXO count in subsector,

p = proportion of total sector investigated,

S = area of subsector investigated,

$\hat{U}$  = expected UXO count in total sector,

$\rho$  = UXO density (per acre) in subsector,

$\rho_s$  = expected UXO density (per acre) in total sector.

**Modules 2-4:** The main probability result is the following:

$$P(U \leq N) = \sum_{j=k}^N \binom{N}{j} p^{j+1} (1-p)^{N-j} \quad (5)$$

where,

$k$  = UXO count in subsector,

$N$  = specified UXO count in total sector,

$p$  = proportion of total sector investigated,

$U$  = actual (generally unknown) UXO count in total sector.

The corresponding UXO density for the specified UXO count is given by:

$$\rho_N = \frac{N}{A} \quad (6)$$

where,

$A$  = area of total sector,

$N$  = specified UXO count in total sector,

$\rho_N$  = UXO density (per acre) corresponding to specified UXO count.

Note that in Module 2,  $N$  is specified ( $A$  is provided in Module 1), while  $\rho_N$  and  $P(U \leq N)$  are output. In Module 3,  $\rho_N$  is specified, while  $N$  and  $P(U \leq N)$  are output. In Module 4,  $P(U \leq N)$  is specified, while  $N$  and  $\rho_N$  are output.

**Module 5:** Finally, the minimum detection level (when no UXO have been found) is given by:

$$P(k \geq 1) = 1 - (1 - p)^N \quad (7)$$

where,

$k$  = UXO count in subsector,

$N$  = specified UXO count in total sector,

$p$  = proportion of total sector investigated.

## Appendix B: UXO Stat Case 1 Hand Calculations

In this example, an area of 500 acres within a sector of area 5000 acres is investigated. No UXO items were found.

**Module 1:** The first four equations are “accounting” calculations for determining UXO counts and densities:

$$p = \frac{S}{A} = \frac{500}{5000} = 0.10 \quad (\text{ref: 1})$$

$$\rho = \frac{k}{S} = \frac{0}{500} = 0 \quad (\text{ref: 2})$$

$$\hat{U} = \frac{k+1}{p} - 1 = \frac{0+1}{0.1} - 1 = 9 \quad (\text{ref: 3})$$

$$\rho_s = \frac{\hat{U}}{A} = \frac{9}{5000} = 1.8E-03 \quad (\text{ref: 4})$$

where,

A = 5000 acres = area of total sector,

k = 0 = UXO count in subsector,

p = proportion of total sector investigated,

S = 500 acres = area of subsector investigated,

$\hat{U}$  = expected UXO count in total sector,

$\rho$  = UXO density (per acre) in subsector,

$\rho_s$  = expected UXO density (per acre) in total sector.

**Module 2:** The main probability result is the following:

$$P(U \leq N) = \sum_{j=k}^N \binom{j}{k} p^{k+1} (1-p)^{j-k} =$$

$$P(U \leq 5) = \sum_{j=0}^5 \binom{j}{0} (0.1)^{0+1} (1-0.1)^{j-0} = \quad (\text{ref: 5})$$

$$0.1[(0.9)^0 + (0.9)^1 + (0.9)^2 + (0.9)^3 + (0.9)^4 + (0.9)^5] =$$

$$0.1[1 + 0.9 + 0.81 + 0.729 + 0.656 + 0.590] = 0.469$$

where,

$k = 0$  = UXO count in subsector,

$N = 5$  = specified UXO count in total sector,

$p = 0.1$  = proportion of total sector investigated,

$U$  = actual (generally unknown) UXO count in total sector.

The corresponding UXO density for the specified UXO count is given by:

$$\rho_N = \frac{N}{A} = \frac{5}{5000} = 1.0E-03 \quad (\text{ref: 6})$$

where,

$A = 500$  acres = area of total sector,

$N = 5$  = specified UXO count in total sector,

$\rho_N$  = UXO density (per acre) corresponding to specified UXO count.

Suppose that in **Module 3**,  $\rho_N$  is specified as  $6.0E-04$  UXO/acre. Then  $N$  may be calculated from equation 6 as  $(6.0E-04) 5000 = 3$  UXO. Then  $P(U \leq N)$  may be calculated as:

$$P(U \leq N) = \sum_{j=k}^N \binom{j}{k} p^{k+1} (1-p)^{j-k} =$$

$$P(U \leq 3) = \sum_{j=0}^3 \binom{j}{0} (0.1)^{0+1} (1-0.1)^{j-0} = \quad (\text{ref: 5})$$

$$0.1[(0.9)^0 + (0.9)^1 + (0.9)^2 + (0.9)^3] =$$

$$0.1[1 + 0.9 + 0.81 + 0.729] = 0.344$$



In **Module 4**, suppose that  $P(U \leq N)$  is specified as 0.19. Then  $N$  may be calculated as that value such that  $P(U \leq N) = 0.19$ . Upon iteration, it is seen that  $N = 1$ , since:

$$\begin{aligned}
 P(U \leq N) &= \sum_{j=k}^N \binom{j}{k} p^{k+1} (1-p)^{j-k} = \\
 P(U \leq 1) &= \sum_{j=0}^1 \binom{j}{0} (0.1)^{0+1} (1-0.1)^{j-0} = \quad \text{(ref: 5)} \\
 0.1[(0.9)^0 + (0.9)^1] &= \\
 0.1[1 + 0.9] &= 0.19
 \end{aligned}$$

Also,  $\rho_N$  may be calculated as:

$$\rho_N = \frac{N}{A} = \frac{1}{5000} = 2.0E-04 \quad \text{(ref: 6)}$$

**Module 5:** Finally, the minimum detection level (when no UXO have been found) is given by:

$$\begin{aligned}
 P(k \geq 1) &= 1 - (1-p)^N \\
 &= 1 - (1-0.1)^5 = 0.4095 \quad \text{(ref: 7)}
 \end{aligned}$$

where,

$k$  = UXO count in subsector,

$N = 5$  = specified UXO count in total sector,

$p = 0.1$  = proportion of total sector investigated.

## Appendix C: UXO Stat Case 2 Hand Calculations

In this example, an area of 400 acres within a sector of area 800 acres is investigated. Twelve UXO items are found.

**Module 1:** The first four equations are “accounting” calculations for determining UXO counts and densities:

$$p = \frac{S}{A} = \frac{400}{800} = 0.5 \quad (\text{ref: 1})$$

$$\rho = \frac{k}{S} = \frac{12}{400} = 0.03 \quad (\text{ref: 2})$$

$$\hat{U} = \frac{k+1}{p} - 1 = \frac{12+1}{0.5} - 1 = 25 \quad (\text{ref: 3})$$

$$\rho_s = \frac{\hat{U}}{A} = \frac{25}{800} = 0.0313 \quad (\text{ref: 4})$$

where,

A = 800 acres = area of total sector,

k = 12 = UXO count in subsector,

p = proportion of total sector investigated,

S = 400 acres = area of subsector investigated,

$\hat{U}$  = expected UXO count in total sector,

$\rho$  = UXO density (per acre) in subsector,

$\rho_s$  = expected UXO density (per acre) in total sector.

**Modules 2:** The main probability result is the following:

$$P(U \leq N) = \sum_{j=k}^N \binom{j}{k} p^{k+1} (1-p)^{j-k} =$$

$$P(U \leq 15) = \sum_{j=12}^{15} \binom{j}{12} (0.5)^{12+1} (1-0.5)^{j-12} =$$

(ref: 5)

$$\binom{12}{12} (0.5)^{13} (0.5)^0 + \binom{13}{12} (0.5)^{13} (0.5)^1 + \binom{14}{12} (0.5)^{13} (0.5)^2 + \binom{15}{12} (0.5)^{13} (0.5)^3 =$$

$$1.22E-04 + 7.93E-04 + 2.78E-03 + 6.94E-03 = 1.06E-02$$

where,

k = 12 = UXO count in subsector,

N = 15 = specified UXO count in total sector,

p = 0.5 = proportion of total sector investigated,

U = actual (generally unknown) UXO count in total sector.

The corresponding UXO density for the specified UXO count is given by:

$$\rho_N = \frac{N}{A} = \frac{15}{800} = 1.88E-02$$

(ref: 6)

where,

A = 800 acres = area of total sector,

N = 15 = specified UXO count in total sector,

$\rho_N$  = UXO density (per acre) corresponding to specified UXO count.

Suppose that in **Module 3**,  $\rho_N$  is specified as 1.63E-02 UXO/acre. Then N may be calculated from equation 6 as (1.63E-02) 800 = 13 UXO. Then  $P(U \leq N)$  may be calculated as:

$$P(U \leq N) = \sum_{j=k}^N \binom{j}{k} p^{k+1} (1-p)^{j-k} =$$

$$P(U \leq 13) = \sum_{j=12}^{13} \binom{j}{12} (0.5)^{12+1} (1-0.5)^{j-12} =$$

(ref: 5)

$$\binom{12}{12} (0.5)^{13} (0.5)^0 + \binom{13}{12} (0.5)^{13} (0.5)^1 =$$

$$1.22E-04 + 7.93E-04 = 9.15E-04$$

In **Module 4**, suppose that  $P(U \leq N)$  is specified as 0.80. Then  $N$  may be calculated as that value such that  $P(U \leq N) = 0.80$ . Upon iteration, it is seen that  $N = 29$ , since:

$$\begin{aligned}
 P(U \leq N) &= \sum_{j=k}^N \binom{j}{k} p^{k+1} (1-p)^{j-k} = \\
 P(U \leq 29) &= \sum_{j=12}^{29} \binom{j}{12} (0.5)^{12+1} (1-0.5)^{j-12} = \\
 &\binom{12}{12} (0.5)^{13} (0.5)^0 + \binom{13}{12} (0.5)^{13} (0.5)^1 + \dots + \binom{28}{12} (0.5)^{13} (0.5)^{16} \quad (\text{ref: 5}) \\
 &+ \binom{29}{12} (0.5)^{13} (0.5)^{17} = 1.22E-04 + 7.93E-04 + \dots + 5.67E-02 \\
 &+ 4.83E-02 = 0.80
 \end{aligned}$$

Also,  $\rho_N$  may be calculated as:

$$\rho_N = \frac{N}{A} = \frac{40}{5000} = 8.0E-03 \quad (\text{ref: 6})$$

**Module 5:** Finally, the minimum detection level (when no UXO have been found) is given by:

$$P(k \geq 1) = 1 - (1-p)^N = 1 - (1-0.5)^{15} = 0.9999694 \quad (\text{ref: 7})$$

where,

$k$  = UXO count in subsector,

$N = 15$  = specified UXO count in total sector,

$p = 0.5$  = proportion of total sector investigated.